Lesson Title and Identifier:
Title: Sea Perch: Density and Center of Mass
Identifier: 4.1

Module: Sea Perch

Subject Area (Unit): Robotics

Concept:
  Density
  Center of Mass

Objectives:
In this module, students will generate information based on their own answers and present this information.

PA Academic Standards:

**Science**
3.1.7 Unifying Themes
3.2.7 Inquiry and Design

**Math**
2.3 Measurement and Estimation
2.9 Geometry

Grade Level: 6th

Setting/Group Size: 12 students

Duration/Time Required: 2 class periods

Materials List (include safety equipment if applicable)

String
Object cutouts
Pencil
Pen with cap
Paper
Ruler
Rings (Gold & Clay)
Beaker
Water
Calculator (Optional)
• Context
  Include background for Teachers
  and background for Students

• Methods and Procedure:
  This includes Pre-Investigative Work (if any)
  Investigative Work
    What evidence do students collect?
    How do students present evidence?
    What conclusions are drawn from the evidence?
  Include example of data sheets

There are several assumptions that are needed for this process,
  • The thickness of the objects are uniform
  • The material in each portion of the object is uniform. (Same density)

Since density can be an abstract concept, we’ll follow Archimedes’ example. Example that density is calculated by the following formula,

\[
density = \frac{mass}{volume}
\]

Use everyday items, like a ring. Have one ring of gold or other metal material and one of play dough. Find the volume of the objects. Drop them in water and measure the change in height. Correlate this to change in volume. Weigh both rings and find the density.

<table>
<thead>
<tr>
<th></th>
<th>Height</th>
<th>Area</th>
<th>Volume</th>
<th>Mass</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold ring</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Clay ring</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To drive the concepts of center of mass and uniform density and their importance, have the students balance a pen to see the effect of adding weight. First tap the cap and the pen. Point out that the cap is hollow while the pen has material inside, namely the ink and the pen nib. In the first case, the students will balance a pen without a cap on their finger. Have the students mark the pen. This will be the control line. Have the students place the cap one end and then the other as in the figure below. Repeat balancing the pen on the same finger. Mark the pen accordingly. Measure the distance from the control line to the new marks. Repeat with a second type of pen. Open for discuss about what happened and what this means for the sub.
To determine the center of mass for an object, there are three methods which could be used,
1) Calculus (Opportunity to lay out what math courses they should be taking to get ready for a career in engineering.)
2) Composite Areas
3) String

Since calculus is beyond the scope of the students’ skill set, we will use this

Procedure:
For the following process, there will be a number of values generated. To keep them in order and to avoid mistakes, you’ll want to use the included tables.
Step 1: Define a coordinate system for the object. You may want to draw it directly on the object.
Step 2: Divide the object into separate portions. Find the area of each portion. Make sure the areas are geometric figures that are easy for the students to work with such as squares, circles, and triangles.

Step 3: Find the center of each area. Determine the X and Y coordinates.

Step 4: Multiply the X coordinate and the area for each portion.

Step 5: Multiply the Y coordinate and the Area for each portion.

Step 6: Add areas of all the portions (Total Area)

Step 7: Add all the values from Step 4

Step 8: Add all the values from Step 5

Step 9: Divide the value from Step 7 by the value in Step 6. This gives you the X coordinate.

Step 10: Divide the value from Step 8 by the value in Step 6. This gives you the Y coordinate.

Step 11: Mark the center of mass on your object. We will verify this value using the String methods.

At the end of this exercise, you may want to point out that they have just completed very basic Mechanics of Materials problem.

Fig. 2: Sample of Object 1. Students should be able to guess where the center of mass is in this sample, but gives them the chance to use their math.
<table>
<thead>
<tr>
<th>Area</th>
<th>( X_1 )</th>
<th>( Y_1 )</th>
<th>( X_1 \times A_1 )</th>
<th>( Y_1 \times A_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area 1</td>
<td>( X_2 )</td>
<td>( Y_2 )</td>
<td>( X_2 \times A_2 )</td>
<td>( Y_2 \times A_2 )</td>
</tr>
<tr>
<td>Area 2</td>
<td>( X_3 )</td>
<td>( Y_3 )</td>
<td>( X_3 \times A_3 )</td>
<td>( Y_3 \times A_3 )</td>
</tr>
<tr>
<td>Area 3</td>
<td>( X_3 \times A_3 )</td>
<td>( Y_3 \times A_3 )</td>
<td>( \text{Total Area} = A_1 + A_2 + A_3 )</td>
<td></td>
</tr>
</tbody>
</table>
| \( \text{Total } X \times \) | \( X_1 \times A_1 \times A_1 + X_2 \times A_2 + X_3 \times A_3 \) | \( \text{Total } Y \times \) | \( Y_1 \times A_1 + Y_2 \times A_2 + Y_3 \times A_3 \) |}

\[ X = \frac{\text{(Total } X \times \text{Area})}{\text{Total Area}} \]

\[ Y = \frac{\text{(Total } Y \times \text{Area})}{\text{Total Area}} \]

**Fig. 3: Sample of Object 2**

**Object 2:**
String Method

Step 1: Create cutouts of the objects.
Step 2: Tape a piece of string to the wall.
Step 3: Add a pin anywhere on the object.
Step 4: Suspend the object using the pin and the string.
Step 5: Draw a line that follows the string.
Step 6: Repeat this two more times, choosing different pin locations.
Step 7: Locate the place where all three line intersect. How far is this from the center of mass they calculated and marked?
Fig. 4: Example of the string technique

Why is this method impractical for the real world?

- Extended Activities:
- Assessment

You will be evaluated on a scale from 0 to 4 on:

<table>
<thead>
<tr>
<th>Participation</th>
<th>Task Completion</th>
<th>Correctness</th>
</tr>
</thead>
</table>

- References & Web Sites:
- Keywords: (up to 10)
- Authors
Connie Gomez and Pat Wilkes

* List any other lessons that this lesson could relate to
  Astronomy
* Attach worksheets
* Be sure to give attribution for any material obtained from other sources