Subject Area: Math

Lesson Title
The Fundamental Theorem of Counting

Grade Level  8

Time Required
Lesson time is dependent on student background, but may require up to 1 hour if the concept of probability has not been previously covered.

Summary
This lesson is intended to be a local application of the probability theory known as the fundamental theorem of counting. The theory can be used to describe the various routes that can be taken from one location to another. A subset of the routes from one location in Philadelphia to another is used for this example.

Keywords
- fundamental theorem of counting
- tree diagram
- branches
- and vs or

Educational Standards
PA Math 2.5.8C, 2.5.8D, 2.7.8A, 2.7.8B

Learning Objectives
After this lesson, students will be able to...

- identify the groups of choices that must be considered
- translate the choices on the map to a tree diagram
- apply the fundamental theorem of counting
- create new questions for this diagram

Introduction / Motivation
Any time more than two choices are made the fundamental theorem of counting may be applied. This theorem is one of the first lessons in probability. Here students are asked to make two simple choices that are familiar to them in an attempt to marry common knowledge with theory.

Associated Activities
Start with a general discussion of the simple map given. Recognize street directions using the compass. Consider the quickest way to get from the university to any location on South Street.

Question 1.
We have three choices for roads that lead East from Drexel university: JFK Boulevard, Market Street and Chestnut Street.

Question 2.
We have four choices for roads that can be used to turn South: 21st Street, 18th Street, Broad Street, and 13th Street.

Question 3.
Let students discuss this question among themselves. This question is the crux of the fundamental theorem of counting. If we have \( a \) options for the first choice and \( b \) options for the second choice, then we will have \( a \times b \) choices overall. Let students discover this relationship in a class discussion. There are 12 overall choices here.

Question 4.
Let the students draw a tree diagram to illustrate all of the choices. There will be three branches for the first set of branches and four branches coming from each of the first three branches.

Question 5.
Start to consider various options. Using the tree diagram, we can see that there is a 4/12 or 1/3 chance that Market Street will be used.

Question 6.
Start to discuss the concept of and vs or. Students will likely need to use the tree diagram to answer this question. There is a 1/3 chance that JFK will be used and ¼ chance that Broad will be used, but the probability of taking both of them is 1/12. Now ask what the probability that either JFK or Broad will be used. There should be 6 routes that use JFK or Broad. 6/12 yields a probability of ½.
Have students construct and answer their own questions until they feel comfortable with the probabilities of taking specific streets or routes.

**Assessment**
This worksheet is designed to inspire thought about the concept of probability rather than to evaluate. Therefore, a separate assessment should be used.

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For a summer job, you have been working with Mr. Steager at his engineering lab at Drexel University. All of the lab members have decided that we want to eat at a restaurant somewhere along South Street. We are going there after work and need to plan the route.

How many choices do we have for roads we can take East?

How many choices do we have for roads we can take South?

How many total different ways can we get there?

Draw a tree diagram representing all of our choices. There will be three branches for the first set of branches and four branches coming from each of the first three branches.

What is the probability that we will take Market Street?

What is the probability that we will take JFK Boulevard?