



Drexel-SDP GK-12 LESSON

Subject Area: Math

Lesson Title

Triangular Numbers

Grade Level 8

Time Required

Lesson time is dependent on student background, but may require up to 1 hour if the concepts of sequences and tessellations have not been previously covered.

Summary

This lesson is intended to finish off a section on triangular numbers. Students should have already been introduced to the concept of sequences and the more specific sequence known as triangular numbers. Note that there is very little existing information at the time of publication on this subject. This example is somewhat involved, but it does show an engineering example of triangular numbers.

Keywords

- sequence
- triangular numbers
- direct formula
- tessellation
- truss
- equilateral

Educational Standards

PA Math 2.3.8A, 2.2.3.8F, 2.4.8D

Learning Objectives

After this lesson, students will be able to...

- identify how a bridge truss may be tessellated
- understand the arithmetic sequence
- apply the concept of sequences to tessellations
- develop an understanding of how triangular numbers may be related to engineering

Introduction / Motivation

Structural trussing is used to support bridges and buildings and is often triangular in shape. This is due to the fact that the triangle is much more rigid under stress than other shapes. Square or rectangular structures tend to fail at the corners when stressed, and circular structures deform into oval shapes. This lesson offers an application of the concept of triangular numbers to bridge trussing.

Associated Activities

The triangular number sequence is as follows: 1, 3, 6, 10, 15, 21, 28... This sequence can be visualized geometrically by tessellating triangles or by imagining stacked cylinders. Each proceeding number in the sequences adds one more than the previous number. 1, 1+2, 1+2+3, 1+2+3+4, 1+2+3+4+5... The direct formula for this sequence is $d = n(n+1)/2$

The worksheet that accompanies this lesson shows a railroad bridge at the top of the page. Students should notice and discuss the geometric patterns they see in the bridge. Next, the worksheet shows a triangle tessellated with 9 smaller triangles. This section is meant to spark a general discussion about triangles, tessellations, and how these tessellations are like the bridge. Note that the top triangle has been grayed out to spark discussion about how the bottom portion of this tessellated triangle looks like a small bridge truss.

The full truss shown at the bottom of the page is meant to spark thought and discussion on the use of triangular number sequences in a larger context with additional logical steps involved. The question is stated, *“Think about how you could use triangular numbers to determine how many meters of steel would be needed to construct this bridge. All of the triangles are equilateral triangles.”*

The following steps can be followed to use triangular number sequences.

- 1) Consider that the side of one smaller triangle is 9 meters/3, or 3 meters.
- 2) Darken or somehow mark the corner triangles and every second triangle in the bridge truss. There should be white triangles between the black, marked triangles.
- 3) Note that there are 9 meters in perimeter for each small, black triangle, and that none of these perimeters overlap.
- 4) Count the number of black or marked triangles along the bottom of the truss. There are 9 total.

5) Use the formula to determine that a 'complete' triangle of this form would yield $d = n(n+1)/2$ black triangles. This is 45 black triangles.

6) Note that the top 'missing' portion of the triangle (where the truss stops) would have 6 black triangles, yielding a total of $6*7/2$ black triangles, or 21 black triangles.

7) Take the difference between these two numbers, $45-21=24$ black triangles left.

8) Multiply by the number of meters per trussed section: $24*9=216$ meters of truss.

9) Note that the top section of the truss would have been subtracted when we 'took away' the top portion of the imaginary large triangle. We have to add this back on. This top measurement is $3*6=18$ meters.

10) The total truss length is $216+18=234$ meters of truss.

Owner

Drexel University GK-12 Program

Contributors

Ed Steager and Cheryl Boyd

Copyright

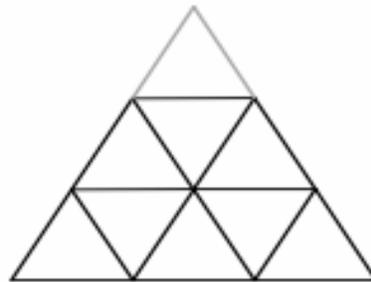
Copyright 2007 Drexel University GK12 Program. Reproduction permission is granted for non-profit educational use.

Engineering a Bridge with Triangular Numbers

Name: _____ Date: _____



The darker bottom portion of this triangle could form the structure of a bridge.



Think about how you could use triangular numbers to determine how many meters of steel would be needed to construct this bridge. All of the triangles are equilateral triangles.

