Drexel-SDP GK-12 LESSON

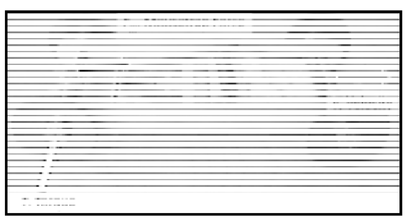
Lesson: Torqued

Subject Area(s) Measurement, Number & Operations, Physical Science, Science & Technology

Associated Unit Forget the Chedda!

Lesson Title Torqued

Header



A force from the mousetrap arm causes a torque on the axle, propelling the car forward.

Grade Level 7 (6-12)

Lesson # 4 of 5

Lesson Dependency

Quantify It; Convert It; Forces, Forces Everywhere

Time Required 20 minutes

Summary

This lesson builds upon the concepts of linear forces and motion to relate the analogous case of torque and rotation. If a net force does not act at an object's center of mass, the object tends to rotate. When these net forces rotate an object, they contribute to a loading known as torque. For the case of the mousetrap-powered car, the torque that causes movement of the axles or wheels is dependent on forces from the mousetrap spring and friction.

Engineering Connection

Torque and torsion are extremely important concepts in Mechanical or Civil engineering and related fields that examine the mechanics or motion of materials or structures. Solid structures not meant for rotation (such as buildings or bridges) may cause bend or twist under torsional loads. The laws of rotational motion can be used to calculate the forces on structures meant to rotate, such as motors, turbines, or axles, and the torque needed to start or maintain rotation. Torque also provides a mechanical advantage that is used by people in all walks of life.

The behavior of stationary and rotating objects must be analyzed when designing efficient and robust structures. Massive windmills used for wind energy have long, rotating blades that must be supported at heights far above the ground. The supporting structure must support the gravitational and bending loads of the moving blades and drag force of the wind. The blades themselves are subject to deflection due to the wind load as they catch and spill wind. All these factors are considered in efficiency and failure analyses of a single windmill before a wind farm is constructed.

Torque has been used for many engineering applications that require heavy lifting because distance magnifies the twisting load due to a force. We call this "magnification" of the load we apply "mechanical advantage." To take advantage of mechanical advantage, wrenches are designed with a long handle (bigger wrench → bigger handle), and sailboats have long tillers and winches/rachets for tightening the jibsheet (the line attached to the front sail).

The mousetrap cars used in this unit can be designed in a similar manner: we can analyze the forces acting on the axle of the drive wheels, which rotates in order to move the car. If there is too much torque acting on the axle, it can be attenuated by lengthening the mousetrap arm (less force is required to wind the arm back → less force is transferred to the rear axle). Choosing a rear axle design that minimizes the moment of inertia will also yield results that optimize the design of the mousetrap cars.

Keywords

torque, force, rotation, acceleration, angular acceleration, moment of inertia, mechanical advantage, moment

Educational Standards

- PA Science:
  - 3.1.7 – Unifying themes
  - 3.2.7.B – Apply process knowledge to make and interpret observations
- PA Math:
  - 2.1.8.D – Apply ratio and proportion to mathematical problem situations involving distance, rate, time, and similar triangles
  - 2.3.5.D – Convert linear measurements within the same system
  - 2.3.8.A – Develop formulas and procedures for determining measurements
  - 2.4.5.B – Use models, number facts, properties, and relationships to check and verify predictions and explain reasoning
  - 2.7.8.D – Compare and contrast results from observations and mathematical models
  - 2.8.8.B – Discover, describe, and generalize patterns, including linear, exponential, and simple quadratic relationships

Pre-Requisite Knowledge

Students must have been introduced to the concept of forces.

Learning Objectives

After this lesson, students should be able to:

- Identify conditions in which torque is used to rotate an object
- Describe what quantities constitute torque
- Compute torque produced by a force acting about a point on an object
- Relate torque and angular acceleration with moment of inertia

Introduction / Motivation

When you built your cars a couple weeks ago, were you disappointed to see that rather than rolling along like you expected, it stayed in place while the wheels spun? When we experimented with friction in our last lesson and activity, we talked about how overcoming static friction may lead to a slipping condition. Would you describe the wheels as slipping on the floor?

So, based on what we found about frictional forces, how can we try to avoid the slipping condition? (Staying within the bounds of the static frictional force → (a) increase surface friction by applying duct tape over wheels or (b) decrease force that pulls axle). Who thinks that if we decrease the force that pulls on the axle, the wheels will turn slower? (Ask for a show of hands; ask if this is their hypothesis). We can explain why a lower force turns the axle at a lower rate by looking at the torque transferred from the force of the mousetrap arm to the axle.

First, however, we should review the angular quantities that we describe rotational position and motion. We think of movement along a line when we think of position, motion, and acceleration and express this in terms of distance (explain while walking at the front of the room, starting and stopping). Who can tell me how we measure angular distance (run a pencil tip along an arc length of a protractor). While we're used to using expressing angles in terms of degrees, we also express angles in terms of radians. Maybe you've seen the "Radian" option when you pressed the "Mode" button on your graphing calculators. There are 2π radians in 360 degrees. So how many radians are in a half circle? (π radians). Can you name some benefits of using radians rather than degrees? (Smaller numbers, fractional representation).

As mentioned earlier, an overall torque on an object results in rotation, specifically angular acceleration. What parallels can you see between the equations F=m*a and T=I*α ? (both equations have a term that relates loading to acceleration with a constant of proportionality determined by or related to mass of the rotating object). How would you apply these equations to your mousetrap car? (F=m*a: frictional force, and T=I*α: torque on the axle from the mousetrap causing the wheels to rotate). If we apply a constant torque to our axle, what will happen? (constant acceleration). Do you think that the size or weight of the rear axle and wheels have an effect on the acceleration? How? (The moment of inertia, I). You can find this out in our related activity (**Spinners**).

Vocabulary / Definitions

Word	Definition
Torque	Loading that causes rotational motion.
Moment of inertia	The degree to which an object will "resist" rotational acceleration when acted upon by a torque.
Linear acceleration	The rate of change of velocity along a linear path.
Angular acceleration	The rate of change of angular velocity.

Lesson Background & Concepts for Teachers

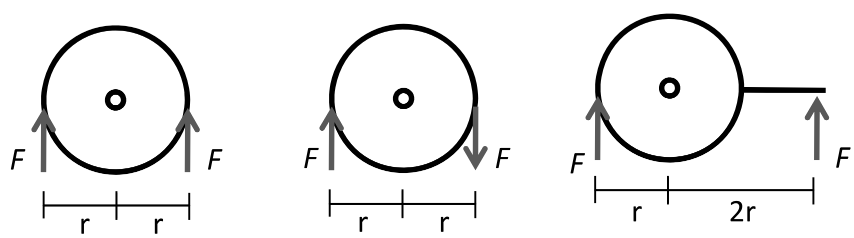
Torsion and torque result from rotational forces applied to an object. In the context of this lesson, torsion is considered to be a rotational force encountered in static equilibrium, when objects are not in motion because all net forces sum to zero. Torque will refer to the rotational (moment) applied to an object that is free to rotate. We can directly apply this lesson to our experimental cars by using the same materials that will rotate. The students have compact discs (CDs) and 1-in diameter washers as part of their building materials, so we will investigate the moment of Inertia of these materials in *Spinners*.

Torque is the rotational analogy to a linear force load. For a rotating object whose is evenly distributed, the torque, T, acting on the object is calculated by multiplying the force, F, by distance to the axis of rotation, d.

$$T = Fd$$

Torque has direction and follows the right-hand rule for rotational assignments: if you point your right thumb in the direction of the axis and curl your fingers in the direction of the net torque, curling your fingers counter-clockwise indicates a *positive moment*, whereas a negative moment is indicated when curling your finders in a clockwise direction.

Torque results from non-zero net moments, which will rotate an object rather than move it linearly. Consider the cases below:



(a) **(b)** **(c)**

diagram (a) shows forces acting in the same direction and at the same distance from the axis of rotation, but on different sides of the axis of rotation. These forces will not rotate the object because the net torque is zero:

$$T_{(a)} = -(Fr) + (Fr) = 0$$

.

Diagram (b) shows a case is similar to diagram (a), but it shows forces acting in opposite directions. These forces rotate the object with a net torque of *T=2Fr*:

$$T_{(b)} = -(Fr) - (Fr) = -2Fr.$$

Diagram (c) shows forces acting in the same direction, on different sides of the object, but at different distances from the axis of rotation. The net torque in this case is *T=Fr*:

$$T_{(c)} = -(Fr) + (F * 2r) = Fr.$$

Inserting numbers in place of the variables *F* and *r* should yield the same results.

Torque is directly proportional to the angular acceleration of an object; that factor of proportionality is the moment of inertia of the rotating object. Angular distance is measured in dimensionless units of degrees or radians; angular velocity, or the change of the angular distance with time, is measured in units of degrees/sec or radians/sec, and is represented by the greek letter omega (lowercase: ω, uppercase: Ω); angular acceleration, or the rate of change of angular velocity with time, is measured in units of degrees/sec^2 or radians/sec^2 and is represented by the greek letter alpha (lowercase: α).

That relation between torque and acceleration can be expressed as: $T = \mathrm{I}\alpha$, where I is the moment of inertia of the object rotating along the same axis as the net moment.

We can relate linear and angular distances and motion by using the idea of circumference, which is essentially the linear distance along the perimeter of a circle. The main relations we should know for this lesson are $v=\omega*r$ and $a=\alpha*r$.

The moment of inertia is analogous as mass is to linear motion: it is a measure of the amount of resistance the object has to motion. The moment of inertia is shape-dependent, and can be simplified to _ for the known shape of a circle. The value of the moment of inertia depends on the position of its mass around its rotational axis. If a circular object has evenly distributed mass, *smaller* circular objects will have *lower* moments of inertia compared to larger circular objects. The end lesson goal of "Spinners" is for students to reach this conclusion.

Note: The technical term for summing the torques is known as summing the *moments* about a point, but to reduce confusion with the temporal meaning of moment, all the forces contributing to twisting loads are called *torques*.

Associated Activities

Spinners

Lesson Closure

Segue into associated activity. Students may need extra time to prepare and review video clips intended for instruction on how to use the PASCO Explorer GLX.

Assessment

Check DataStudio **Spinners** workbook: is there a calculated value for I?

Additional Multimedia Support

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Owner

Drexel University GK-12 Program

Contributors

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