Drexel-SDP GK-12 LESSON

Lesson: Collisions

Subject Area(s) Data Analysis & Probability, Measurement, Number & Operations, Physical Science, Science and Technology

Associated Unit Vital Mechanics

Lesson Title Collisions

Rack 'em.

Grade Level 7 (6-12)

Lesson # 1 of 3

Lesson Dependency

Time Required 20 minutes

Summary

This lesson outlines the physics theory behind collisions. It is recommended that the lesson serve as a wrap-up after students complete the associated activity.

Engineering Connection

Elastic collisions are the cornerstone example of energy transfer, both on the molecular and macroscale levels. While chemistry and thermodynamics topics focus on the molecular level (gas laws, phase changes, etc.), collisions of solid objects are important considerations for those who work in fields involving ballistics, forensics, safety mechanisms, and athletics.

Theorems on Conservation of Energy and Conservation of Momentum provide a framework for analyzing and predicting the motion of particles or objects and forces associated with contact. If we assume that elastic contact occurs between two objects, we expect the energy transfer to result in purely physical motion of the two original objects, not heat or sound. Conversely, a plastic collision implies that the two objects changed somehow (think of clay or Play-Doh balling up). Use of both theorems allows us to equate the initial and final energy states of the system.

These concepts are used extensively in the design of sporting equipment (baseball bats, racket sports, golf balls, protective padding) and research and development for automotive safety products (seat belts, air bags) and military/aerospace applications.

Keywords

mass, velocity, momentum, kinetic energy, potential energy

Educational Standards

- PA Science:
    - 3.1.7 – Unifying themes
    - 3.2.7.B – Apply process knowledge to make and interpret observations
- PA Math:
    - 2.1.8.D – Apply ratio and proportion to mathematical problem situations involving distance, rate, time, and similar triangles
    - 2.4.5.B – Use models, number facts, properties, and relationships to check and verify predictions and explain reasoning
    - 2.5.8.B – Verify and interpret results using precise mathematical language, notation and representations, including numerical tables and equations, simple algebraic equations and formulas, charts, graphs, and diagrams
    - 2.7.8.D – Compare and contrast results from observations and mathematical models
    - 2.8.8.B – Discover, describe, and generalize patterns, including linear, exponential, and simple quadratic relationships

Pre-Requisite Knowledge

Students should know how to measure with a meter stick and a protractor.

Learning Objectives

After this lesson, students should be able to:

- Relate velocity to ramp position and height (Conservation of Energy)
- Discuss the effect of mass and velocity on momentum
- Discuss the effect of angles on ramp height and 2-D particle velocity
- Predict the outcome of a collision with knowledge of particle momentum (1-D and 2-D Conservation of Linear Momentum)

Introduction / Motivation [use as wrap up – guided class discussion]

Let's review what happened in our activity. If you were following the activity sheet, you should have started things off by investigating head-on collisions. What happened if you used the same type of ball? *(balls should collide and come to rest)* What about different combinations – golf vs. foam, foam vs. ping pong? *(heavier ball will roll through other one, minimally deflected; lighter ball will deflect backwards)*. What do you think would happen if the same types of balls collided but one had more velocity (speed) than the other? *(Similar to heavier/lighter scenario: ball with more speed will roll through one with less).*

How does the collision angle change when we rotate the angle of the ramp and vary the velocity? *(This changes the velocity of the balls involved in the collision. Increasing the speed of the balls is another way to change the velocity. For each ramp angle, different velocities will result in different final collision angles).*

What did you find out about oblique (sideswipe) impacts? *(An oblique impact results in both balls deflecting 90^o from their original paths – final paths are 180^o to each other)*.

How could these concepts be used? *(Car accident reconstruction, billiards/pool, ballistics, etc.)*. Based on what you found out today, do you think that a lighter ball might be able to push a heavier ball out of its way? *(With enough velocity, yes)*.

Vocabulary / Definitions

Word	Definition
Mass	A measure of matter packed into an object. Equal to weight divided by gravity.
Velocity	The speed and direction of a particle. In a 2-D representation of particle motion, each direction is defined by a magnitude which adds up to the overall speed.
Momentum	The quantity defined as the product of mass and velocity of an object.
Collision	Impact of one object with another.
Oblique	Relating to the side of an object. An *oblique* collision is referred to as a "sideswipe."
Potential Energy	Energy possessed by an object before it goes into motion. Springs and gravity are sources of potential energy.
Kinetic	Energy possessed by an object in motion, calculated as $\frac{1}{2}mv^2$, which is half of

Energy	the product of the mass and square of the velocity.

Lesson Background & Concepts for Teachers

This module introduces the concept of collisions to students in a hands-on demonstration and can be scaled accordingly for a range of grade levels. Head-on collisions are referred to as 1-D (one-dimensional) since the resulting motion is typically along a line. Rotating the ramp makes the problem 2-D (two-dimensional) and requires the use of algebra and trigonometry for quantitative assessment and comparison to theory (Conservation of Momentum).

Since the module's intended audience did not cover algebraic systems of equations or trigonometry, the focus in this lesson is on the role of angles in collisions. Students can make qualitative comparisons of variations in momentum at each collision angle. This is accomplished by setting the ramps at equal angles (using wooden blocks) and then setting balls at different positions on the ramp. With respect to the mathematics behind the physics, the velocity of the ball at the end of the ramp can be found using the Conservation of Energy theory. This states that the sum of Potential Energy (PE) and Kinetic Energy (KE) is the same in a given system. For us, setting "the system" consists of two particles on ramps that collide. The initial height (h) of the ball can be found from the angle of the ramp (θ) and the distance the ball is placed from the end (l),

$$h = l \sin\theta .$$

The starting energy is potential energy – which results from gravity (g=9.81 m/s^2) acting on the initial height of the ball,

$$\text{PE} = mgh .$$

If we release the ball from rest (zero velocity → zero KE) and it rolls down the ramp, all PE is transferred into KE, so the energy relation from the initial to final states is

$$\text{KE}_{\text{initial}} + \text{PE}_{\text{initial}} = \text{KE}_{\text{final}} + \text{PE}_{\text{final}}$$

Kinetic energy was defined as $\frac{1}{2} mv^2$, where v is velocity.

$$\left(\tfrac{1}{2} mv^2\right)_{\text{initial}} + \left(mgh\right)_{\text{initial}} = \left(\tfrac{1}{2} mv^2\right)_{\text{final}} + \left(mgh\right)_{\text{final}}$$

For one ball on one ramp, our initial velocity is zero and the final height is zero (the floor). The first and fourth terms in the equation above drop out, and the mass of the ball cancels if the ball doesn't change in mass while it is traveling down the ramp.

$$gh_{\text{initial}} = \tfrac{1}{2} v_{\text{final}}^2$$

And the final velocity (which is the speed in 1-D cases) can be calculated from the initial height,

$$v_{\text{final}} = \sqrt{2gh_{\text{initial}}} = \sqrt{2gl \sin\theta} .$$

So we see that the angle θ of the ramp has an effect on the final velocity v_{final} of the ball at a given placement l. This is a brief summary of the equations meant to show the direct relation of PE and KE, and consequently the relation between height and velocity of a falling particle.

Conservation of momentum ($L=mv$) can be defined similarly, but for a system with two particles we use the subscripts 1 and 2 to identify their masses and velocities,

$$L_{\text{initial}} = L_{\text{final}}$$

$$m_1 v_{1,\text{initial}} + m_2 v_{2,\text{initial}} = m_1 v_{1,\text{final}} + m_2 v_{2,\text{final}}\,.$$

A head-on collision between a moving and nonmoving particle is straightforward. Since the speed of the moving particle is equal to the final velocity from the ramp v_{final}, we use that as the initial velocity v_{initial} in the momentum equation,

$$m_1 v_{1,\text{initial}} = m_1 v_{1,\text{final}} + m_2 v_{2,\text{final}}\,.$$

We can use the Conservation of Energy principle for before and after the collision for our other relation between initial and final velocities. Again, keeping $v_{2,\text{initial}}=0$,

$$\left(\tfrac{1}{2} mv^2\right)_{1,\text{initial}} = \left(\tfrac{1}{2} mv^2\right)_{1,\text{final}} + \left(\tfrac{1}{2} mv^2\right)_{2,\text{final}}\,.$$

The system of equations can then be solved using substitution. If we have two moving particles colliding head-on, we still have an additional term in each equation, and the steps are similar.

However, if we solve the system of equations for two particles colliding at an angle, then we must use the full definition of *velocity* (speed and direction) and use trigonometry to map out the *component* velocities (separate velocities in different directions). The angles measured in the activity are used to find the component velocities. The momentum and energy equations also apply for each direction (*i.e.* 4 total equations and 4 total unknowns).

Associated Activities

Ramped Up

Lesson Closure

Wrap-up with this lesson / discussion.

Owner

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