Drexel-SDP GK-12 ACTIVITY

Activity Template

Subject Area(s): Sound

Associated Unit:

Associated Lesson: None

Activity Title: What is the quickest way to my destination?

Grade Level: 8 (7-9)

Activity Dependency: None

Time Required: 90 minutes

Group Size: 3-4 students

Expendable Cost per Group: US $0.00

Summary:
In this activity, students will explore basic concepts of graph theory and determine the quickest route from one location to another using simple graph theory algorithms. As a result, the students will learn how mathematical relationships can be turned into graphs to solve problems more efficiently. This activity thus gives the students an overview of graph theory, applications of graph theory, and the fundamental components of a graph.

Engineering Connection:
Engineers working with graph theory attempt to model real-world relationships with a graph that portrays each entity and the connections between all pairs of entities. Engineers working in the field of chemistry and physics often use graph theory when studying molecules. The graph theory structure works well for molecules since atoms can be represented as nodes and bonds between atoms can be represented as edges connecting the nodes. Similarly, engineers studying computer networks use graph theory to model the relationship between servers and network traffic using nodes and edges, respectively. Overall, graph theory has strong connections with various types engineering applications that involve modeling systems or structures.

Keywords: graph theory, nodes, edges, system modeling

Educational Standards

Science: 3.1 – Unifying Themes
Math: 2.4 – Mathematical Reasoning and Connections
2.5 – Mathematical Problem Solving and Communication

Learning Objectives

After this lesson, students should be able to:

- Explain the basic idea of graph theory, using nodes and edges to model a system
- Define a node and give examples of what a node could represent in a graph
- Define an edge and give examples of what an edge could represent in a graph
- Provide an example of how graph theory applies to everyday life

Materials List

Each group needs:

- worksheet
- calculator

Introduction / Motivation

Many systems and structures in the real world have multiple interrelated objects that can be easily modeled and visualized using graph theory. This generally works well for engineers attempting to gain a better understanding of a certain behavior in science or to assist in solving a complex mathematical problem. In general, graph theory problems are categorized based on the goal of algorithms applied to the graph or by the modeling method used to generate the graph. Specific to this activity, the students explore Shortest Path Graphs that are characterized by an algorithm applied to the graph that attempts to find the lowest sum of edges when traveling from the beginning node to the goal node. Although students explore a very simple example of graph theory in this activity, engineers use far more complex graphs to simplify the complexity of extremely convoluted systems in order to find reasonable solutions to problems.

Vocabulary / Definitions

Word	Definition
node	a fundamental unit in graph theory that represents an entity or object
edge	a connection between a pair of nodes that relates to the nodes in some manner

Procedure

Background

Graph theory is a defined as the study of graphs, which can be defined as visual representations of objects in a system where there may exist relationships between pairs of objects. The use of graph theory allows for the creation of a visual representation of a complex problem by modeling a given system, thereby easing the process of problem solving. A graph consists of two components, nodes and edges, which represent objects and relationships between the objects, respectively. In a graph, nodes and edges may be airports with possible flights connecting the airports, servers in network with network traffic connecting the servers, or atoms in a molecule connected by bonds holding the molecule together. There exist many types of graphs with edges that may be undirected (two-way street), directed (one-way street), mixed (both two-way and one-way streets), or various other types. In this activity, students will explore undirected graphs since they are more simple and easier to understand. The study of graph theory is quite large and more background can be acquired if desired, but the basic concepts are all that is need for this activity.

Before the Activity

- Print out a copy of the lab for each student
- Prepare the supplies for each group (calculator and copy of the worksheet)

With the Students

1. Ask the students if they have ever heard of any graphs other than those that are used to plot data.
2. Explain to the students that their job is to determine the shortest path between two airports using various types of graph theory algorithms.
3. Explain to the students that once they understand graph theory and the basic fundamentals they will be able to solve similar problems by modeling systems.
4. Provide the students with a general background of graph theory that can be taken directly from the Background section.
5. Create an example graph that students can relate to given your knowledge of the students
6. Dedicate a substantial amount of time to explaining the purpose of nodes and edges so that students can later apply this knowledge to the worksheet problems.
7. In the example, go through the three types of algorithms presented in the worksheet, namely closest neighbor, visiting least number of nodes, and combinatorial search.
8. Provide the students with the handout and have them work together in small groups of preferably three students, but no more than four students.
9. Have the students check with you periodically to see if their answers are correct since the answers are provided below.
10. Wrap up by re-explaining graph theory and how it can be applied to other real world problems.

Attachments

- Graph Theory Handout
- Graph Theory Handout Answers

Safety Issues
None

Troubleshooting Tips
None

Investigating Questions
- Do graphs allow you to create relationships between each and every node?
- When would you want to use directed graphs where the edges only allow traffic in one direction?

Assessment

Pre-Activity Assessment
Class Discussion:
- Talk with students about graph theory and the many applications of graph theory.
- After explaining the idea of graph theory see if students can think of applications where graph theory could be applied.

Activity Embedded Assessment
Lab handout/worksheet: Have the students fill out the lab handout and review their answers as a measure of the concept comprehension.

Post-Activity Assessment
Graph Creation: Have the students create their own graph that is a shortest path graph and create a few problems for another student in the class to complete.

Activity Extensions
- Have the students explore other types of graphs in the graph theory field.
- Have the students research a new type of graph and create their own graph.

Activity Scaling
- For lower grades, just have the students complete the first three or four problems on the worksheet.
- For upper grades, the weights can be modified or more cities can be added to the graph. Also, new algorithms can be created for the problems.

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Graph Theory Handout

Names: _____

Airport Codes – Airport Name (City, State)

LAX – Los Angeles International Airport (Los Angeles, California)
SEA – Seattle/Tacoma International Airport (Seattle, Washington)
LAS – Mccarran International Airport (Las Vegas, Nevada)
DEN – Denver International Airport (Denver, Colorado)
HOU – Hobby Airport (Houston, Texas)
ATL – Atlanta International Airport (Atlanta, Georgia)
ORD – Chicago O'hare International Airport (Chicago, Illinois)
IAD – Washington Dulles International Airport (Washington, D.C.)
JFK – John F. Kennedy International Airport (New York, New York)
MIA – Miami International Airport (Miami, Florida)

Determining the shortest route to the destination

In each situation below you are to determine the shortest route between your current location and the destination city. The possible routes are shown in green on the map with the travelling distances listed on the route lines. Depending on the current situation, some airports may be closed due to inclement weather or mechanical problems. In these cases, you will need to find an alternate route that reaches the destination city.

1. You need to get from your business headquarters near LAX to meet with a business partner at JFK. Find the shortest travel distance between LAX and JFK. Make sure you list each airport visited during the trip and the total number of miles traveled.

2. There is a problem at LAS with the lights on the runway and all planes are being diverted from LAS. You must adjust your route in problem 1 so that it does not travel to LAS. Make sure you list each airport visited during the trip and the total number of miles traveled.

3. ATL just radioed in that airplanes cannot land because of bad weather conditions and the lights on the runway at LAS still do not work. You must adjust your route in problem 2 so that it does not travel to LAS or ATL. Make sure you list each airport visited during the trip and the total number of miles traveled.

4. Now that you met with your business partner it is time to get back to headquarters in LAX. You are not ready to get back to work yet so you are trying to find the longest route back to LAX, but each flight must always travel west (i.e., IAD to MIA is considered traveling west because the line is slanted towards the left). Make sure you list each airport visited during the trip and the total number of miles traveled. NOTE: LAS has fixed the runway lights and the weather has cleared up at ATL, so both airports are accepting arrivals.

Determining the route to the destination with the first specified algorithm

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As the number of cities and connections between cities increase the number of possible travel routes increases at a much faster rate. Therefore, engineers that solve similar problems will use algorithms that limit the number of travel routes by searching in an "intelligent" manner. A few basic algorithms will be discussed in the following problems.

The first algorithm you will use finds the closest airport to the east of your current location until reaching the destination airport without re-visiting any airports. Do you expect this algorithm to provide the shortest route to the destination airport, why or why not?

5. We are going to repeat problem 1 with the new algorithm. You need to get from your business headquarters near LAX to meet with a business partner at JFK. Find the shortest travel distance between LAX and JFK using the new algorithm. Make sure you list each airport visited during the trip and the total number of miles traveled.

6. We are going to repeat problems 2 and 3 combined with the new algorithm. ATL just radioed in that airplanes cannot land because of bad weather conditions and the lights on the runway at LAS still do not work. You must adjust your route in problem 5 so that it does not travel to LAS or ATL while still using the new algorithm. Make sure you list each airport visited during the trip and the total number of miles traveled.

7. We are going to repeat problem 4 with the new algorithm. Now that you met with your business partner it is time to get back to headquarters in LAX. You are not ready to get back to work yet so you are trying to find the longest route back to LAX. The only restriction is that each flight must end further west than the departing airport (i.e., IAD to MIA is considered traveling west because the line is slanted towards the left). The algorithm should now be modified to find the airport that is farthest from your current airport location. Make sure you list each airport visited during the trip and the total number of miles traveled. NOTE: LAS has fixed the runway lights and the weather has cleared up at ATL, both airports are accepting arrivals.

Determining the route to the destination with the second specified algorithm

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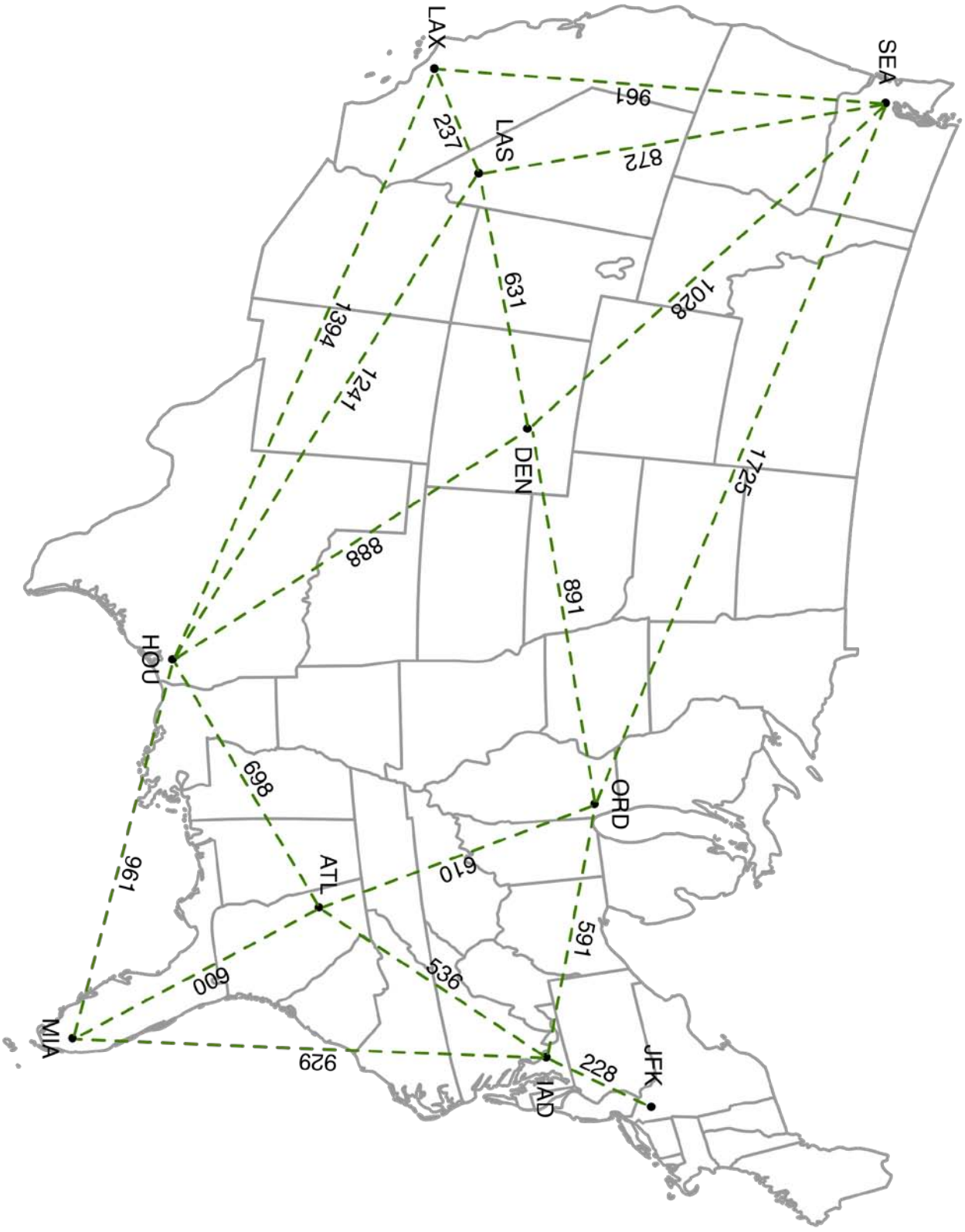
The second algorithm you will use attempts to get from the current location to the destination city in the shortest number of stops. The only restriction is that each arrival city must be east of the departure city. Do you expect this algorithm to provide the best or shortest route to the destination airport, why or why not?

8. We are going to repeat problem 1 with the second algorithm. You need to get from your business headquarters near LAX to meet with a business partner at JFK. Find the route between LAX and JFK that visits the least number of other airports. Make sure you list each airport visited during the trip and the total number of miles traveled. NOTE: Total mileage does not matter.

9. ATL just radioed in that airplanes cannot land because of bad weather conditions and the lights on the runways at LAS and HOU do not work. Find a route in the shortest number of stops that reaches the destination without traveling to LAS, HOU or ATL. Make sure you list each airport visited during the trip and the total number of miles traveled.

10. We are going to repeat problem 4 with the second algorithm. Now that you met with your business partner it is time to get back to headquarters in LAX. You are not ready to get back to work yet so you are trying to find the longest route back to LAX that visits the most number of airports, but each flight must always travel west (i.e., IAD to MIA is considered traveling west because the line is slanted towards the left). Make sure you list each airport visited during the trip and the total number of miles traveled. NOTE: LAS and HOU have fixed the runway lights and the weather has cleared up at ATL, all airports are accepting arrivals.

Airport Map


LAX
SEA
LAS
DEN
HOU
ORD
ATL
MIA
JFK
IAD
961
237
872
631
1028
1394
1241
1725
888
891
698
961
610
600
536
591
929
228

Graph Theory Handout Answers

Names: _____

Airport Codes – Airport Name (City, State)

LAX – Los Angeles International Airport (Los Angeles, California)
SEA – Seattle/Tacoma International Airport (Seattle, Washington)
LAS – Mccarran International Airport (Las Vegas, Nevada)
DEN – Denver International Airport (Denver, Colorado)
HOU – Hobby Airport (Houston, Texas)
ATL – Atlanta International Airport (Atlanta, Georgia)
ORD – Chicago O'hare International Airport (Chicago, Illinois)
IAD – Washington Dulles International Airport (Washington, D.C.)
JFK – John F. Kennedy International Airport (New York, New York)
MIA – Miami International Airport (Miami, Florida)

Determining the shortest route to the destination

In each situation below you are to determine the shortest route between your current location and the destination city. The possible routes are shown in green on the map with the travelling distances listed on the route lines. Depending on the current situation, some airports may be closed due to inclement weather or mechanical problems. In these cases, you will need to find an alternate route that reaches the destination city.

1. You need to get from your business headquarters near LAX to meet with a business partner at JFK. Find the shortest travel distance between LAX and JFK. Make sure you list each airport visited during the trip and the total number of miles traveled.

LAX to LAS:	237 mi
LAS to DEN:	631 mi
DEN to ORD:	891 mi
ORD to IAD:	591 mi
IAD to JFK:	228 mi
Total	2578 mi

2. There is a problem at LAS with the lights on the runway and all planes are being diverted from LAS. You must adjust your route in problem 1 so that it does not travel to LAS. Make sure you list each airport visited during the trip and the total number of miles traveled.

LAX to HOU:	1394 mi
HOU to ATL:	698 mi
ATL to IAD:	536 mi
IAD to JFK:	228 mi
Total	2856 mi

3. ATL just radioed in that airplanes cannot land because of bad weather conditions and the lights on the runway at LAS still do not work. You must adjust your route in problem 2 so that it does not travel to LAS or ATL. Make sure you list each airport visited during the trip and the total number of miles traveled.

LAX to HOU:	1394 mi
HOU to MIA:	961 mi
MIA to IAD:	929 mi
IAD to JFK:	228 mi
Total	3512 mi

4. Now that you met with your business partner it is time to get back to headquarters in LAX. You are not ready to get back to work yet so you are trying to find the longest route back to LAX, but each flight must always travel west (i.e., IAD to MIA is considered traveling west because the line is slanted towards the left). Make sure you list each airport visited during the trip and the total number of miles traveled. NOTE: LAS has fixed the runway lights and the weather has cleared up at ATL, both airports are accepting arrivals.

JFK to IAD:	228 mi
IAD to MIA:	929 mi
MIA to ATL:	600 mi
ATL to HOU:	698 mi
HOU to DEN:	888 mi
DEN to LAS:	631 mi
LAS to SEA:	872 mi
SEA to LAX:	961 mi
Total	5807 mi

Determining the route to the destination with the first specified algorithm

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As the number of cities and connections between cities increase the number of possible travel routes increases at a much faster. Therefore, engineers that solve similar problems will use algorithms that limit the number of travel routes by searching in an "intelligent" manner. A few basic algorithms will be discussed in the following problems.

The first algorithm you will use finds the closest airport to the east of your current location until reaching the destination airport without re-visiting any airports. Do you expect this algorithm to provide the shortest route to the destination airport, why or why not?

I do not agree because it doesn't guarantee that you will find the shortest route. Many examples exist to show this.

5. We are going to repeat problem 1 with the new algorithm. You need to get from your business headquarters near LAX to meet with a business partner at JFK. Find the shortest travel distance between LAX and JFK using the new algorithm. Make sure you list each airport visited during the trip and the total number of miles traveled.

LAX to LAS:	237 mi
LAS to DEN:	631 mi
DEN to HOU:	888 mi
HOU to ATL:	698 mi
ATL to IAD:	536 mi
IAD to JFK:	228 mi
Total	3218 mi

6. We are going to repeat problems 2 and 3 combined with the new algorithm. ATL just radioed in that airplanes cannot land because of bad weather conditions and the lights on the runway at LAS still do not work. You must adjust your route in problem 5 so that it does not travel to LAS or ATL while still using the new algorithm. Make sure you list each airport visited during the trip and the total number of miles traveled.

LAX to SEA:	961 mi
SEA to DEN:	1028 mi
DEN to HOU:	888 mi
HOU to MIA:	961 mi
MIA to IAD:	929 mi
IAD to JFK:	228 mi
Total	4995 mi

7. We are going to repeat problem 4 with the new algorithm. Now that you met with your business partner it is time to get back to headquarters in LAX. You are not ready to get back to work yet so you are trying to find the longest route back to LAX. The only restriction is that each flight must end further west than the departing airport (i.e., IAD to MIA is considered traveling west because the line is slanted towards the left). The algorithm should now be modified to find the airport that is farthest from your current airport location. Make sure you list each airport visited during the trip and the total number of miles traveled. NOTE: LAS has fixed the runway lights and the weather has cleared up at ATL, both airports are accepting arrivals.

JFK to IAD:	228 mi
IAD to MIA:	929 mi
MIA to HOU:	961 mi
HOU to LAX:	1394 mi
Total	3512 mi

Determining the route to the destination with the second specified algorithm

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The second algorithm you will use attempts to get from the current location to the destination city in the shortest number of stops. The only restriction is that each arrival city must be east of the departure city. Do you expect this algorithm to provide the best or shortest route to the destination airport, why or why not?
No because it doesn't guarantee to be the shortest route.

8. We are going to repeat problem 1 with the second algorithm. You need to get from your business headquarters near LAX to meet with a business partner at JFK. Find the route between LAX and JFK that visits the least number of other airports. Make sure you list each airport visited during the trip and the total number of miles traveled. NOTE: Total mileage does not matter.

OPTION #1

LAX to SEA:	961 mi
SEA to ORD:	1725 mi
ORD to IAD:	591 mi
IAD to JFK:	228 mi
Total	3505 mi

OPTION #2

LAX to HOU:	1394 mi
HOU to ATL:	698 mi
ATL to IAD:	536 mi
IAD to JFK:	228 mi
Total	2856 mi

OPTION #3

LAX to HOU:	1394 mi
HOU to MIA:	961 mi
MIA to IAD:	929 mi
IAD to JFK:	228 mi
Total	3512 mi

9. ATL just radioed in that airplanes cannot land because of bad weather conditions and the lights on the runways at LAS and HOU do not work. Find a route in the shortest number of stops that reaches the destination without traveling to LAS, HOU or ATL. Make sure you list each airport visited during the trip and the total number of miles traveled.

LAX to SEA:	961 mi
SEA to ORD:	1725 mi
ORD to IDA:	591 mi
IAD to JFK:	228 mi
Total	3505 mi

10. We are going to repeat problem 4 with the second algorithm. Now that you met with your business partner it is time to get back to headquarters in LAX. You are not ready to get back to work yet so you are trying to find the longest route back to LAX that visits the most number of airports, but each flight must always travel west (i.e., IAD to MIA is considered traveling west because the line is slanted towards the left). Make sure you list each airport visited during the trip and the total number of miles traveled. NOTE: LAS and HOU have fixed the runway lights and the weather has cleared up at ATL, all airports are accepting arrivals.

OPTION #1

JFK to IAD:	228 mi
IAD to MIA:	929 mi
MIA to ATL:	600 mi
ATL to HOU:	698 mi
HOU to DEN:	888 mi
DEN to LAS:	631 mi
LAS to SEA:	872 mi
SEA to LAX:	961 mi
Total	5807 mi

OPTION #2

JFK to IAD:	228 mi
IAD to MIA:	929 mi
MIA to ATL:	600 mi
ATL to ORD:	610 mi
ORD to DEN:	891 mi
DEN to LAS:	631 mi
LAS to SEA:	872 mi
SEA to LAX:	961 mi
Total	5722 mi