Drexel-SDP GK-12 ACTIVITY

Activity Template

Subject Area(s): Probability

Associated Unit: None

Associated Lesson: None

Activity Title: Probability, Permutations and Combinations

Grade Level: 8 (7-9)

Activity Dependency: None

Time Required: 90 minutes

Group Size: 3-4 students

Expendable Cost per Group: US $0.50

Summary:
In this activity, students explore probability with the help of playing cards while relating permutations and combinations to the calculation of the probability. As the students experiment with playing cards the students will build an appreciation of probability in everything from leisure activities to professional careers. Thus proving to students that probability is not only a component in a mathematics career, but a part of everyday life.

Engineering Connection:
Engineers specializing in digital signal processing (DSP) often use probability when developing classifications algorithms, radar systems and motion detection systems. In identification algorithms, engineers use probability to determine who or what is the most likely candidate. For example, a voice identification system would listen to a voice and try to identify the individual based on the likelihood (probability) of the voice matching a specific person's model.

Keywords: probability, permutations, combinations, factorial

Educational Standards
- Science: None
- Math: 2.7 – Probability and Predictions

Learning Objectives
After this lesson, students should be able to:
- Explain and give an example of a real-world situation that involves probability
- Define probability, permutation, combination and factorial
- Calculate probability, number of permutations, number of combinations and factorials

Materials List
Each group needs:
- 13 playing cards at the least

To share with the entire class:
- None

Introduction / Motivation
People participate in activities and use device on a daily basis without realizing its involvement in the study of probability. For example, when most people leave to go somewhere they pick up their cell phone to take along with them, but do they think about the failure rates of the hardware components in the phone. Most likely the answer is no. When hardware components are being developed they are rigorously tested to determine the durability and life of the component. The results of these tests are presented in terms of probabilities, such as the failure rate of a component. If a developer takes a new circuit board to a company to be promote the use of the new board in a device and the circuit board has a failure rate of 50% then no company would even think of using such a device. In general, the probabilities of all types of events are calculated in order for people to make the best products, life choices, safety restrictions and financial decisions in every part of the world on a daily basis.

Vocabulary / Definitions

Word	Definition
probability	ratio of the number of favorable occurrences to the total number of occurrences
permutation	an ordered sequence of numbers from a list of numbers
combination	an unordered sequence of numbers from a list of numbers

factorial	the product of all positive integers less than or equal to n (denoted n!)

Procedure
Background
The use of probability, permutations and combinations are used daily in many different fields that range from gambling to mechanical failure rates to rates of detection in medical screening. Calculating the probability of an outcome uses the definitions below:

Probability: The formula below calculates the likelihood of an event occurring by dividing the number of times an event could occur by the total number of events that could occur.

$$\text{Probability ratio} = \text{\# of favorable outcomes} / \text{\# of outcomes}$$

Permutation: The formula below calculates the number of permutations of a given set of data where n is the total number of elements in the data set and k is the number of elements selected at one time. Notice that the definition of a permutation states that the ordering of the element selection does matter.

$$\text{\# of permutations} = \frac{n!}{(n-k)!}$$

Combination: The formula below calculates the number of combinations of a given set of data where n is the total number of elements in the data set and k is the number of elements selected at one time. Notice that the definition of a combination states that the ordering of the element selection does not matter.

$$\text{\# of combinations} = \frac{n!}{k!(n-k)!}$$

Factorial: The formula below calculates the factorial of a given number where n is the number.

$$n! = \prod_{k=1}^{n} k \quad \forall n \in \mathbb{N}$$

Before the Activity
- Print out a copy of the handout for each student
- Divide the decks of cards into stacks of about 13 or more for each group

With the Students
1. Divide the students into groups of three to four students
2. Give each group a copy of the handout
3. Define each of the four terms at the top of the handout and give the students a mathematical example of each (i.e., playing cards)
4. Make sure the students know all of the cards in a deck by writing the card numbers/letters and suits on the board (i.e., 2H, QD, 5C, JS)
5. Have the students complete the both sections *Card Experiment – Probability* and *Card Experiment – Permutations and Combinations* in groups
6. After the students complete this section, show them the formula for calculating the number of permutations and combinations for a given data set along with an example

7. Have the students complete both sections *Calculations with formulas* and *Challenge Calculations with formulas*
8. Pass out a stack of cards to each group for the next section (If no cards are available then randomly assign students 13 different cards on a piece of paper using just numbers/letters with the suit)
9. Have the students complete the *Probability Calculations* section
10. Collect the worksheets before the students leave the classroom

Attachments
- [Probability, Combinations and Permutations Handout](#)

Safety Issues
- None

Troubleshooting Tips
There are no common issues with this activity.

Investigating Questions
- What situations or games involve both probability and permutations or combinations?
- How does someone determine if they are to find the number of combinations or permutations in a given situation?

Assessment

Pre-Activity Assessment
Class Discussion:
- Talk with the students about different instances where knowing the probability of a situation is important and ask if they can come up with any situations where probability is important
- Talk with the students about different careers that use probability on a daily basis and explain what could happen in instances of incorrect probability calculations

Activity Embedded Assessment
Handout: Have the students fill out the handout and review their answers as a measure of the concept comprehension

Post-Activity Assessment
Worksheet/homework: Create a worksheet with various different scenarios of cards and have the students calculate different probabilities similar to those in *Probability Calculations* section of the handout.

Activity Extensions

- Students could experiment with situations that involve probability, permutations and combinations such as dice
- Students could study a card game such as blackjack and determine the probability of winning the game based on the cards known

Activity Scaling
- For lower grades, the cards for each group should be the exact same so that students can be walked through the process to find the answers
- For upper grades, a full deck of cards could be given to each group or three dice could substitute for the cards with a different set of questions

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Probability, Combinations and Permutations

Names: _____

Definitions
_____

Probability –

Permutation –

Combination –

Factorial –

Card Experiment – Probability
_____

1. Find the probability of drawing a card that is red with an even number from a full deck of cards.

2. Find the probability of drawing a card that black without a number from a full deck of cards.

Card Experiment – Permutations and Combinations
_____

1. List the permutations and combinations of selecting two cards from a set of five cards. Find the number of permutations and combinations.

Permutations List	Combinations List
Number of Permutations = _____	Number of Combinations = _____

2. List the permutations and combinations of selecting three cards from a set of five cards. Find the number of permutations and combinations.

Permutations List | Combinations List

Number of Permutations = _____ | Number of Combinations = _____

Formulas:

Permutation | Combination

Calculations with formulas:

3. Using the formulas we just stated, determine the number of permutations and combinations for when we select 5 cards from a set of 20 cards. **Show your work!**

4. Using the formulas we just stated, determine the number of permutations and combinations for when we select 5 cards from a set of 35 cards. **Show your work!**

Challenge Calculations with formulas:

5. Determine the number of cards to select from a set of 10 cards to obtain the largest number of permutations. In other words, find the value of k that yields the most number of permutations.

6. Determine the number of cards to select from a set of 10 cards to obtain the largest number of combinations. In other words, find the value of k that yields the most number of combinations.

Probability Calculations:

7. List the cards in your set of cards, such as 7♦, 2♣, J♠, A♥, etc.

8. Find the probability of first picking a card with a club and then picking a card without a number on it, replacing the cards in the stack.

9. Find the probability of first picking a card that is red suit and then picking a card that is red suit, without replacing the cards in the stack.

10. Find the probability of first picking a card with a number on it and then picking a card without a number on it, without replacing the cards in the stack. (There are two answers!)

11. Find the probability of first picking a card with an even number on it and then picking a card with an odd number on it, replacing the cards in the stack.

12. Find the probability of first picking a card with a number less than 8 and then picking a card with a number greater than 5, without replacing the cards in the stack. (There are two answers!)